BAYESIAN ESTIMATION OF POPULATION PROPORTION OF A STIGMATIZED ATTRIBUTE USING A FAMILY OF ALTERNATIVE BETA PRIORS

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Abstract

In this study, we have developed the Bayes estimators of the population proportion of a stigmatized attribute when data were gathered through the randomized response technique (RRT) put forward by Hussain and Shabbir [9]. Using both the Kumaraswamy (KUMA) and the Generalised (GLS) beta distributions as a family of alternative beta priors, superiority of the derived Bayes estimators was established for a large interval of the values of the population proportion. We observed that for small, moderate as well as large sample sizes, the alternative Bayes estimators were better than the Bayes estimator proposed by Hussain and Shabbir [10] when a simple beta prior was used.

***Keywords**: Alternative Bayes estimators (ABEs), a family of alternative beta priors (FABPs), Stigmatized attribute, Mean Square Error (MSE), Absolute Bias.*

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1. Introduction

Direct interrogation about a stigmatized attribute such as induced abortion, use of drug, tax evasion, etc. in a human population survey is a difficult exercise. A sampler may receive wrong answers from the survey respondents when he/she uses direct interrogating approach. Due to many reasons, information about prevalence of stigmatized attributes, in the population, is essential. Warner [24] was the first to propose a complicated method of survey to gather information in relation to stigmatized attributes by ensuring confidentiality and anonymity to the respondents. Up till now, a vast number of developments and improvements on Warner's Randomized Response Technique (RRT) have been developed by several researchers. Greenberg et al. [8], Mangat and Singh [16], Mangat [15], Singh et al. [21], Christofides [7], Kim and Warde [14], Adebola and Adepetun [2], Adebola and Adepetun [3], Adepetun and Adebola [4] are some of the many to be listed. In some situations, prior information about the unknown parameter may be available and can be used along with the sample auxiliary information for determination of that unknown parameter known as the Bayesian approach of estimation. Work done by researchers on Bayesian analysis of Randomized response models are not very much, nevertheless, attempts have been made on the Bayesian analysis of Randomized response techniques. Winkler and Franklin [25], Pitz [20], Spurrier and Padgett [22], O'Hagan [18], Oh [19], Migon and Tachibana [17], Unnikrishnan and Kunte [23], Bar-Lev and Bobovich [5], Barabesi and Marcheselli [6], Kim et al. [13], Hussain and Shabbir [10,11], Hussain and Shabbir [12], Adepetun and Adewara [1], are the major references on the Bayesian analysis of the Randomized Response Techniques. The paper is arranged as follows. In Section 2, we present Hussain and Shabbir [10] Randomized Response Technique (RRT) followed by our proposed alternative Bayesian estimation of population proportion in section 3. Section 4 contains the numerical consideration and comparison of results while section 5 is the conclusion.

2. The Existing Bayesian Technique of Estimation

Hussain and Shabbir [10] in their referred paper presented a Bayesian estimation to the Randomized Response Technique (RRT) proposed by Hussain and Shabbir [9] using a simple beta prior distribution to estimate the population proportion of respondents possessing stigmatized attribute.
Assume the simple beta prior is defined as follows

$$f(\pi) = \frac{1}{B(a,b)}\pi^{a-1}(1-\pi)^{b-1}\ ;\ 0<\pi<1 \tag{1}$$

where (a,b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the stigmatized attribute.

Let $X = \sum x_i$ be the total number of the yes response in a sample of size n selected from the population with simple random sampling with replacement sampling. Then the conditional distribution of X given π is

$$f(X|\pi) = \frac{n!}{x!\,(n-x)!}\phi^x(1-\phi)^{n-x} \tag{2}$$

where ϕ is the probability of "yes response" to the stigmatized attribute which is defined as

$$\phi = \frac{\alpha}{\alpha+\beta}\big(P_1\pi + (1-P_1)(1-\pi)\big) + \frac{\beta}{\alpha+\beta}\big(P_2\pi + (1-P_2)(1-\pi)\big) \tag{3}$$

where P_1 is the preset probability of "yes" response to the stigmatized attribute and (α,β) are non-zero constants such that $P_1 + P_2 = 1$ respectively

$$f(X|\pi) = \binom{n}{x}\left[\frac{\pi\big((2P_1-1)(\alpha-\beta)\big)+\beta P_1+\alpha P_2}{\alpha+\beta}\right]^x\left[1-\frac{\pi\big((2P_1-1)(\alpha-\beta)\big)+\beta P_1+\alpha P_2}{\alpha+\beta}\right]^{n-x}$$

On simplification, we have

$$f(X|\pi) = \binom{n}{x}\left[\frac{\big((2P_1-1)(\alpha-\beta)\big)}{\alpha+\beta}\right]^n(\pi+F)^x(1-\pi+H)^{n-x}$$

where $F = \frac{\beta P_1+\alpha P_2}{(2P_1-1)(\alpha-\beta)}$; $H = \frac{3P_1(\beta-\alpha)+3\alpha}{(2P_1-1)(\alpha-\beta)}$

Letting $A = \binom{n}{x}\left[\frac{\big((2P_1-1)(\alpha-\beta)\big)}{\alpha+\beta}\right]^n$

$$f(X|\pi) = A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i(1-\pi)^j \tag{4}$$

for $x = 0,1,2,\ldots,n$

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Thus, the joint probability density functions (pdf) of X and π is

$$f(X,\pi) = D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{i}\pi^{a-1}(1-\pi)^{j}\,(1-\pi)^{b-1} \quad (5)$$

where $D = \frac{\binom{n}{x}}{B(a,b)}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^{n}$

Now the marginal distribution of X can be obtained by integrating the joint distribution of X and π over π. Thus the marginal distribution of X is given by

$$f(X) = \int_0^1 f_g(X,\pi)\,d\pi = D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1 \pi^{a-1+i}(1-\pi)^{b-1+j}\,d\pi$$

$$f(X) = D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\,B(a+i,b+j) \quad (6)$$

The posterior distribution of π given X is defined as

$$f(\pi|X) = \frac{f(X,\pi)}{f(X)} \quad (7)$$

$$f(\pi|X) = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{a-1+i}(1-\pi)^{b-1+j}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\,B(a+i,b+j)} \quad (8)$$

Under the squared error loss function, the Bayes estimator of π which is the posterior mean of (8) is given by

$$\hat{\pi}_{SH} = \int_0^1 \pi\, f(\pi|X)d\pi \; = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1 \pi^{a+i}(1-\pi)^{b+j-1}\,d\pi}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\,B(a+i,b+j)}$$

$$= \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\,B(a+i+1,b+j)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\,B(a+i,b+j)} \quad (9)$$

The Bias of $\hat{\pi}_{SH}$ as well as its Mean Square Error (MSE) is given by

$$B(\hat{\pi}_{SH}) = \hat{\pi}_{SH} - \pi \quad (10)$$

$$MSE(\hat{\pi}_{SH}) = \sum_{x=0}^{n}(\hat{\pi}_{SH}-\pi)^2\phi^{x}(1-\phi)^{n-x} \quad (11)$$

3. The Proposed Alternative Bayesian Techniques of Estimation

Here, we present an alternative Bayesian estimation to Hussain and Shabbir [9] Randomized Response Technique using both the Kumaraswamy (KUMA) and the Generalised (GLS) beta prior distributions as our family of alternative beta prior distributions in addition to the simple beta prior distribution used by Hussain and Shabbir [10].

3.1.1 Estimation of π using Kumaraswamy prior

The Kumaraswamy prior distribution of π is given as

$$f(\pi) = bc\pi^{c-1}(1-\pi^{c})^{b-1}\;\; ;b,c\; > 0 \quad (12)$$

Using the Kumaraswamy prior in (12), the joint probability density function of X and π is derived as

$$f(X,\pi) = abE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{i}(1-\pi)^{j}\,(1-\pi^{c})^{b-1}\pi^{c-1} \quad (13)$$

where $E = \binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^{n}$

The marginal probability density function (pdf) of X can be obtained as

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$$f(X) = \int_0^1 f(X,\pi)\, d\pi \tag{14}$$

$$= abE \sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} \int_0^1 (1-\pi)^j \pi^{ck+i+c-1} d\pi$$

$$= abE \sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} B(ck+c+i, j+1) \tag{15}$$

Similarly, the posterior distribution as usual is obtained as follows

$$f(\pi|X) = \frac{\sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} (1-\pi)^j \pi^{ck+i+c-1}}{\sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} B(ck+c+i, j+1)} \tag{16}$$

Under the Square error loss, we proceed to obtain the posterior mean which is the Bayes estimator as follows

$$\hat{\pi}_{KH} = \int_0^1 \pi\, f(\pi|X) d\pi \tag{17}$$

Considering the fact that

$$\int_0^1 \pi\, (1-\pi)^j \pi^{ck+i+c-1} d\pi = \int_0^1 \pi^{ck+i+c} (1-\pi)^j d\pi = B(ck+i+c+1, j+1)$$

Therefore,

$$\hat{\pi}_{KH} = \frac{\sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} B(ck+i+c+1, j+1)}{\sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} B(ck+i+c, j+1)} \tag{18}$$

As a result, the Bias of $\hat{\pi}_{KH}$ as well as its Mean Square Error is also given by

$$B(\hat{\pi}_{KH}) = \hat{\pi}_{KH} - \pi \tag{19}$$

$$MSE(\hat{\pi}_{KH}) = \sum_{x=0}^{n} (\hat{\pi}_{KH} - \pi)^2 \phi^x (1-\phi)^{n-x} \tag{20}$$

3.1.2 Estimation of π using Generalised Beta prior

The Generalised Beta prior is defined as

$$f(\pi) = \frac{c}{B(a,b)} \pi^{ac-1} (1-\pi^c)^{b-1}; \quad a,b,c > 0 \tag{21}$$

where a, b, c are the shape parameters of the prior distribution as given in equation (21)

By binomial series expansion, we know that

$$(1-\pi^c)^{b-1} = \sum_{k=0}^{b-1} (-1)^k \binom{b-1}{k} (\pi^c)^k$$

consequently

$$f(\pi) = \frac{c}{B(a,b)} \sum_{k=0}^{b-1} (-1)^k \binom{b-1}{k} \pi^{c(k+a)-1}$$

As a result, the joint density function of π and X with Generalized beta prior is

$$f(X,\pi) = G \sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} (1-\pi)^j \pi^{c(a+k)+i-1} \tag{22}$$

where

$$G = \frac{c}{B(a,b)}\binom{n}{x}\left[\frac{\left((2P_1-1)(\alpha-\beta)\right)}{\alpha+\beta}\right]^n$$

The marginal probability density function (pdf) of X can then be obtained from (22) as

$$f(X) = \int_0^1 f(X,\pi)\,d\pi \tag{23}$$

$$= G\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1) \tag{24}$$

Similarly, we obtained the posterior distribution of π given X as

$$f(\pi|X) = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^j\pi^{c(a+k)+i-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1)} \tag{25}$$

In the same manner, under the square error loss, the posterior mean which is otherwise known as the Bayes estimator is given by

$$\hat{\pi}_{GH} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i+1,j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1)} \tag{26}$$

The Bias of $\hat{\pi}_{GH}$ and its Mean Square Error (MSE) are respectively given by

$$B(\hat{\pi}_{GH}) = \hat{\pi}_{GH} - \pi \tag{27}$$

$$MSE(\hat{\pi}_{GH}) = \sum_{x=0}^{n}(\hat{\pi}_{GH}-\pi)^2\,\phi^x(1-\phi)^{n-x} \tag{28}$$

4. Numerical consideration and comparison of Results

Here, we present the numerical consideration as well as comparative study of our results with the existing Hussain and Shabbir [10]. In order to determine the Absolute Biases and the associated Mean Square Errors (MSEs) of both the conventional and the proposed estimators, we assume the values of the parameters in the priors as beta (a=2,b=3,c=1); Kuma (a=1,b=3,c=4) ;GLS (a=2,b=3,c=4) at various selected sample sizes. To surmount the associated computational complexity and generate these results, we wrote computer programs using R-statistical software. To save a considerable number of spaces, we present few results in tables and figures as follows:

Table 1a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10,\ P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	5.808662E-09	1.812328E-08	2.604882E-08
0.2	2.917126E-09	1.263793E-08	1.937450E-08
0.3	1.011638E-09	8.138637E-09	1.368623E-08
0.4	9.220036E-11	4.625392E-09	8.984007E-09
0.5	1.588118E-10	2.098197E-09	5.267834E-09
0.6	1.211473E-09	5.570505E-10	2.537710E-09
0.7	3.250183E-09	1.953728E-12	7.936349E-10
0.8	6.274943E-09	4.329063E-10	3.560946E-11
0.9	1.028575E-08	1.849908E-09	2.636334E-10

Table 1b- Table showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,\ P_2 = 0.9$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.34324461	0.60629503	0.72687499
0.2	0.24324461	0.50629503	0.62687499
0.3	0.14324461	0.40629503	0.52687499
0.4	0.04324461	0.30629503	0.42687499
0.5	0.05675539	0.20629503	0.32687499
0.6	0.15675539	0.10629503	0.22687499
0.7	0.25675539	0.00629503	0.12687499
0.8	0.35675539	0.09370497	0.02687499
0.9	0.45675539	0.19370497	0.07312501

Figure 1a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10,$ $P_1 = 0.1, P_2 = 0.9$

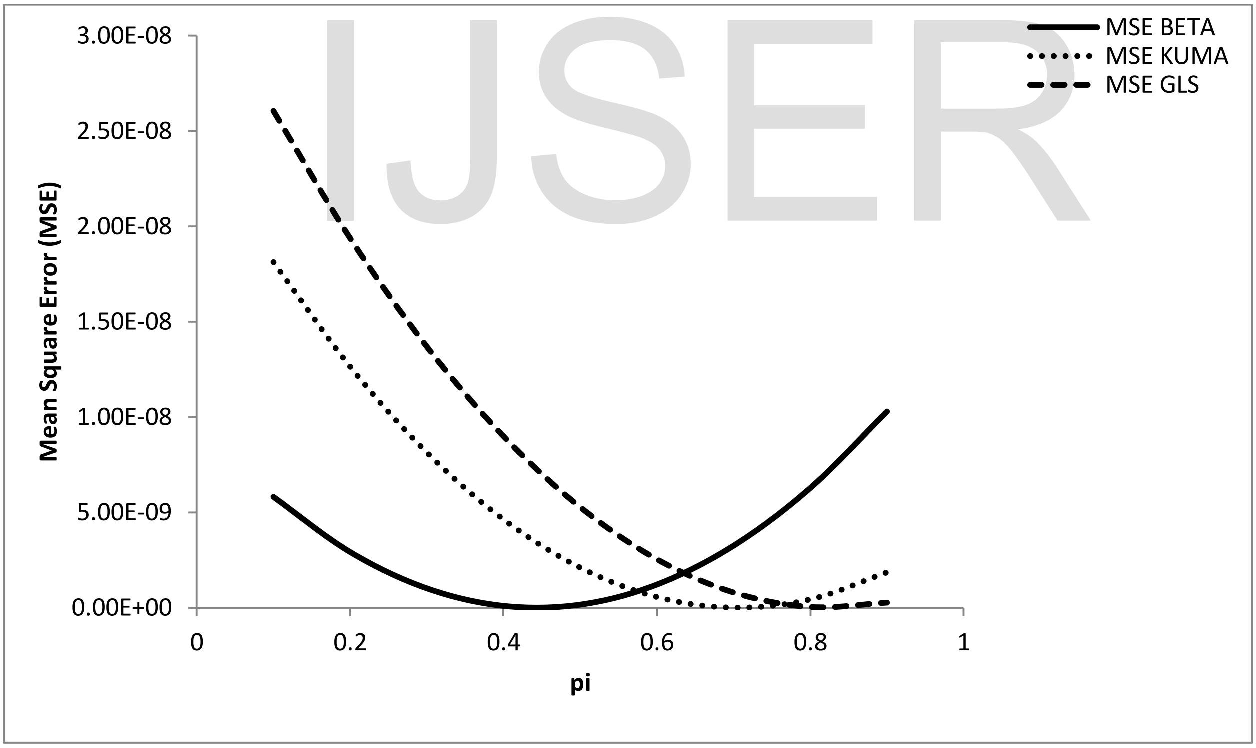


Figure 1b- Graph showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,$ $P_2 = 0.9$

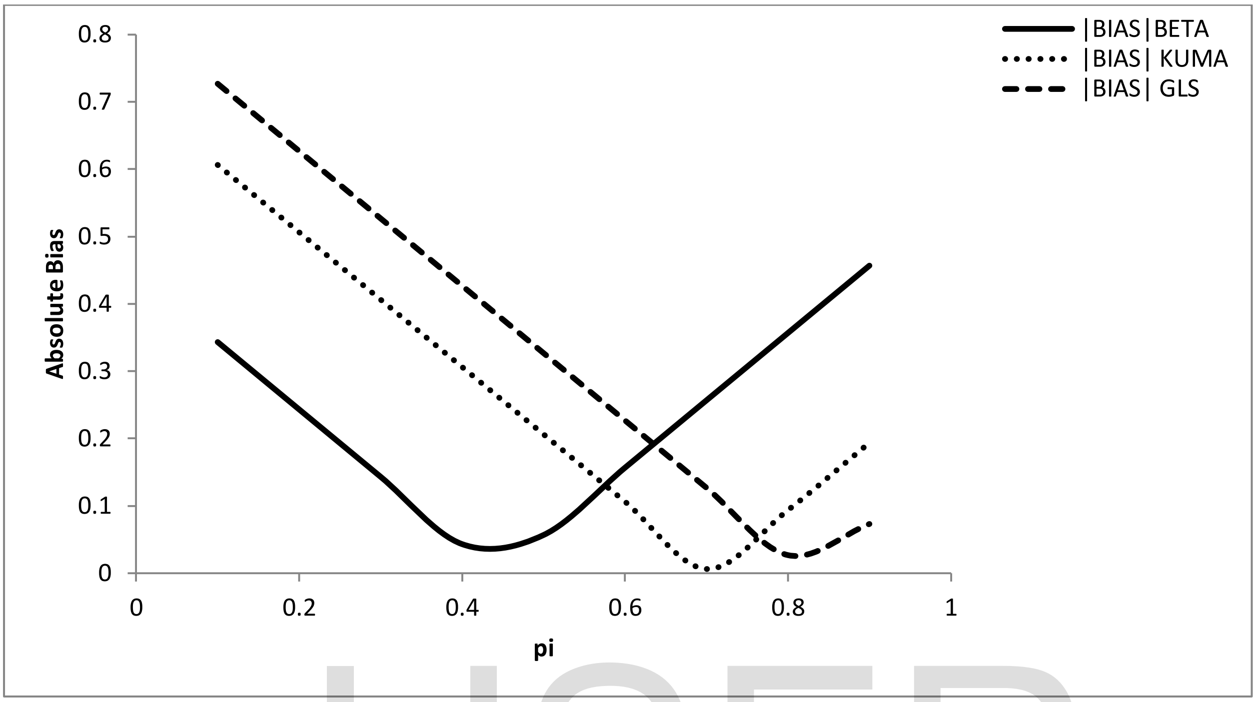


Table 2a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	5.350364E-09	2.139775E-08	2.903802E-08
0.2	2.595090E-09	1.539474E-08	2.196363E-08
0.3	8.258649E-10	1.037778E-08	1.587528E-08
0.4	4.268932E-11	6.346865E-09	1.077298E-08
0.5	2.455631E-10	3.302003E-09	6.656737E-09
0.6	1.434486E-09	1.243190E-09	3.526539E-09
0.7	3.609459E-09	1.704262E-10	1.382391E-09
0.8	6.770481E-09	8.371190E-11	2.242912E-10
0.9	1.091755E-08	9.830470E-10	5.224118E-11

Table 2b- Table showing the Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.32942560	0.65879411	0.76744841
0.2	0.22942560	0.55879411	0.66744841
0.3	0.12942560	0.45879411	0.56744841
0.4	0.02942560	0.35879411	0.46744841
0.5	0.07057440	0.25879411	0.36744841
0.6	0.17057440	0.15879411	0.26744841
0.7	0.27057440	0.05879411	0.16744841
0.8	0.37057440	0.04120589	0.06744841
0.9	0.47057440	0.14120589	0.03255159

Figure 2a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10, P_1 = 0.2, P_2 = 0.8$

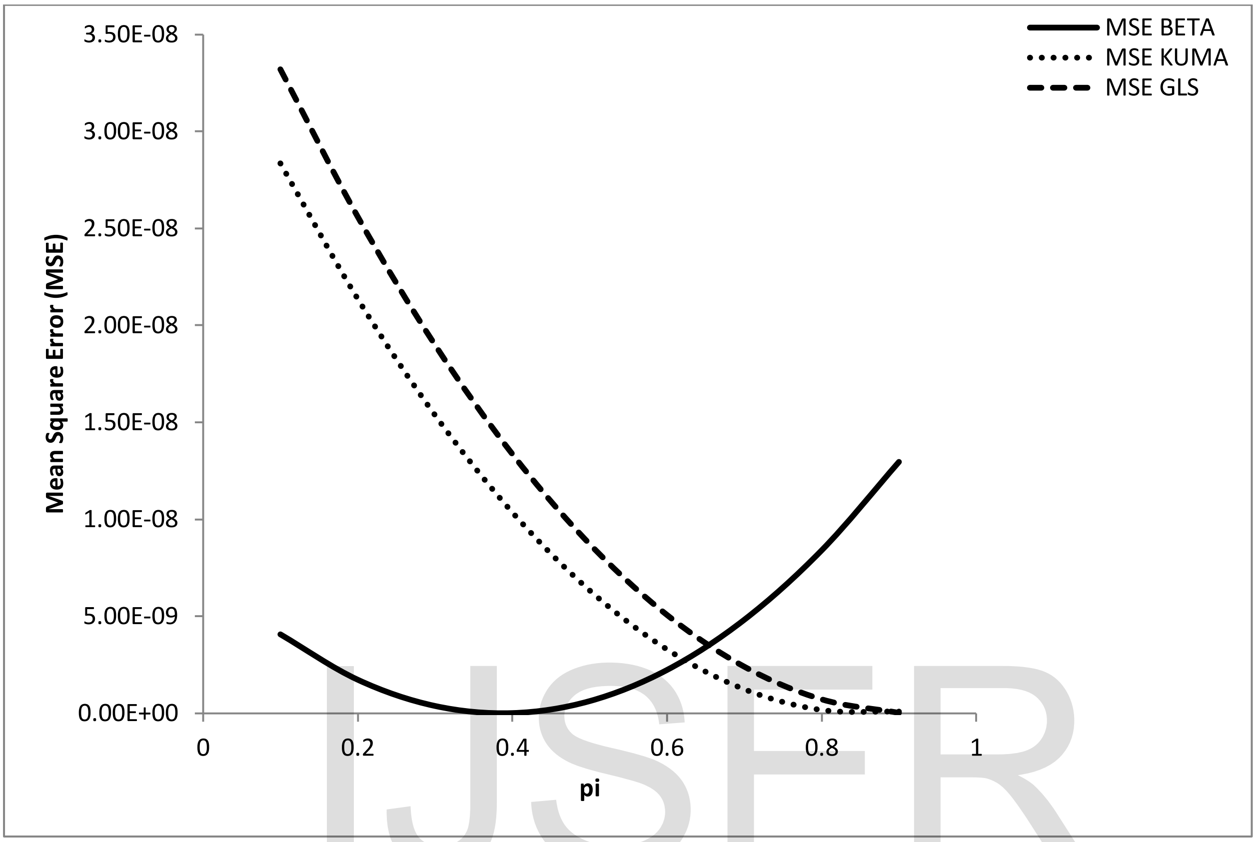
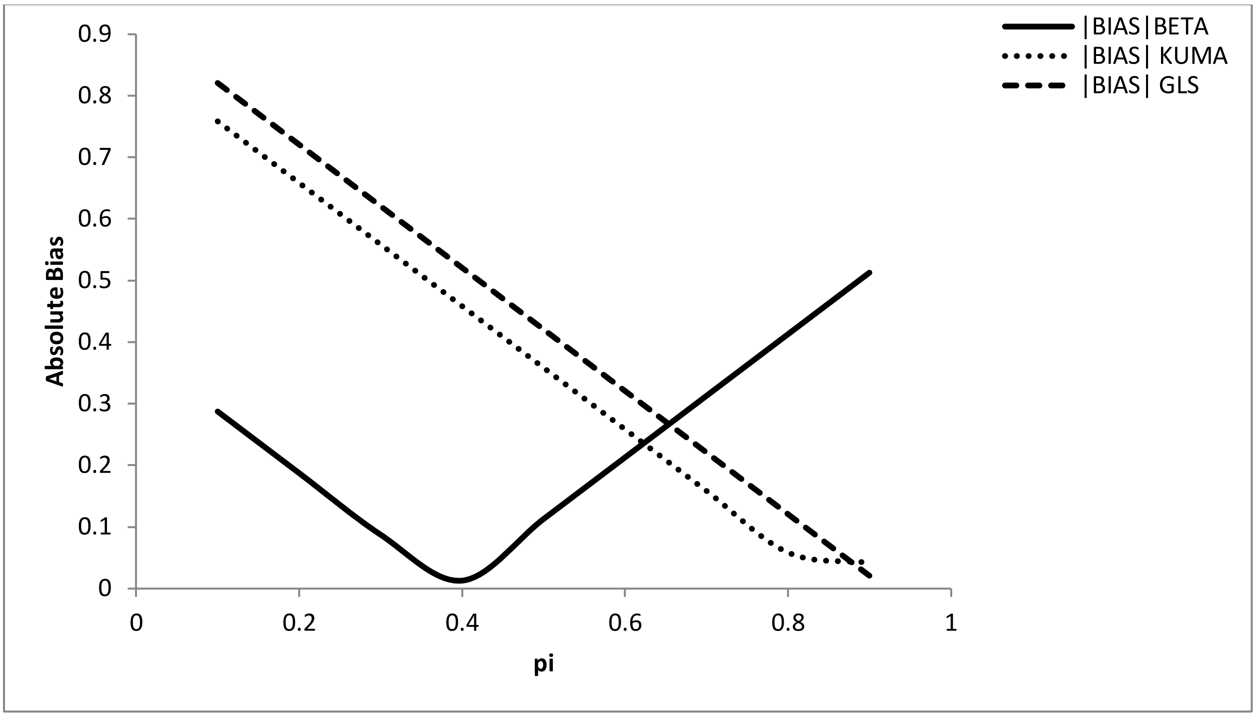


Figure 2b- Graph showing the Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 25, x = 15, \alpha = 1,\ \beta = 10, P_1 = 0.2, P_2 = 0.8$



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Comment: When $n = 25, P_1 = 0.1, 0.2$, the conventional estimator only perform averagely when π lies between 0.1 and 0.5 while the proposed estimators perform significantly better in obtaining higher and truthful responses from respondents when π lies between 0.6 and 0.9 respectively.

Table 3a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 100, x = 60, \alpha = 1, \ \beta = 10, \ P_1 = 0.1, \ P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	8.418124E-31	1.297804E-30	1.718592E-30
0.2	4.548558E-31	8.030642E-31	1.140362E-30
0.3	1.860685E-31	4.264941E-31	6.803017E-31
0.4	3.545060E-32	1.680933E-31	3.384106E-31
0.5	3.001950E-33	2.786176E-32	1.146888E-31
0.6	8.872260E-32	5.799571E-33	9.136300E-33
0.7	2.926126E-31	1.019067E-31	2.175311E-32
0.8	6.146718E-31	3.161831E-31	1.525392E-31
0.9	1.054900E-30	6.486288E-31	4.014946E-31

Table 3b- Table showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 100, x = 60, \alpha = 1, \ \beta = 10, \ P_1 = 0.1, \ P_2 = 0.9$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.37745946	0.46866999	0.53932311
0.2	0.27745946	0.36866999	0.43932311
0.3	0.17745946	0.26866999	0.33932311
0.4	0.07745946	0.16866999	0.23932311
0.5	0.02254054	0.06866999	0.13932311
0.6	0.12254054	0.03133001	0.03932311
0.7	0.22254054	0.13133001	0.06067689
0.8	0.32254054	0.23133001	0.16067689
0.9	0.42254054	0.33133001	0.26067689

Figure 3a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 100, x = 60, \alpha = 1, \ \beta = 10, \ P_1 = 0.1, \ P_2 = 0.9$

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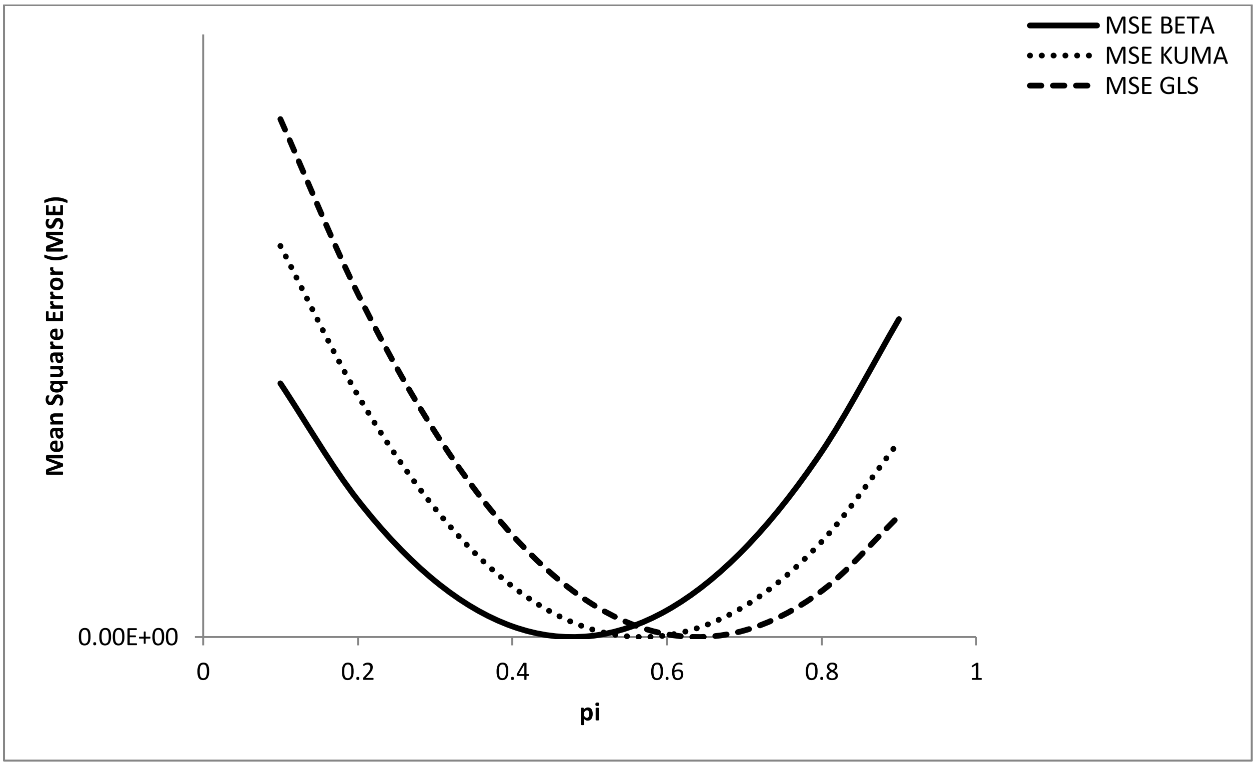

Figure 3b- Graph showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 100, x = 60, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,$ $P_2 = 0.9$

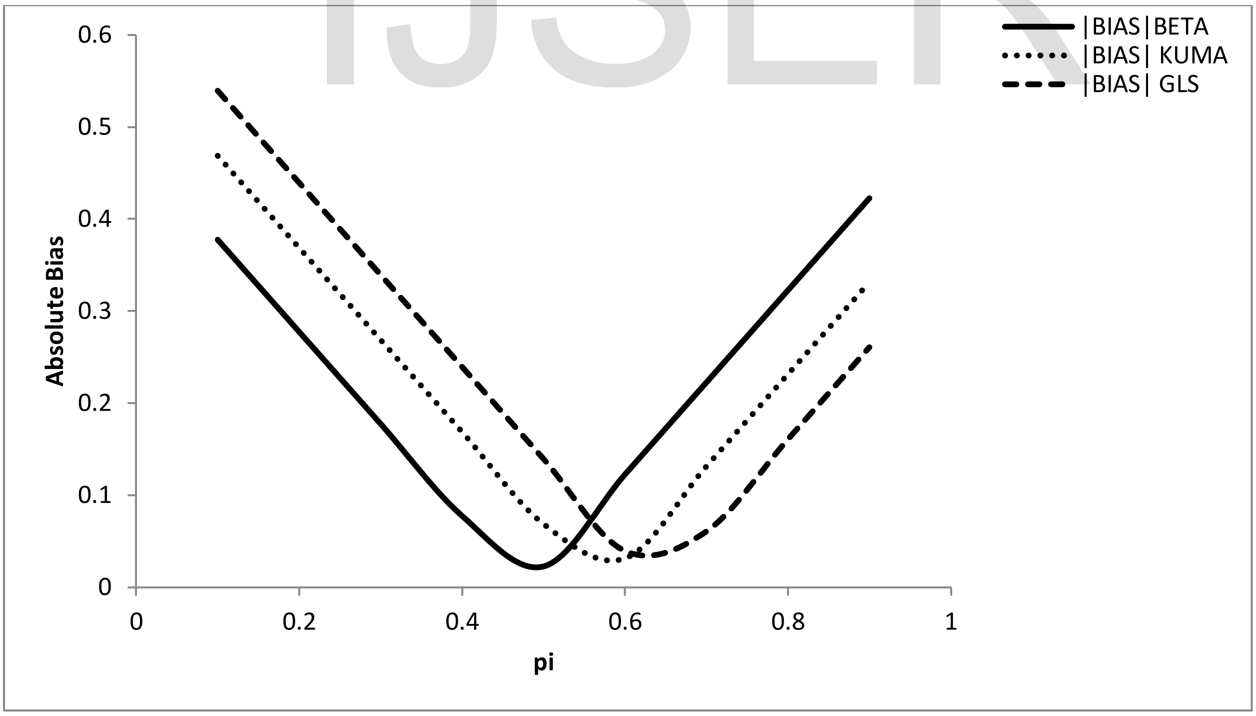

Table 4a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 100, x = 60, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	7.847912E-31	1.550351E-30	2.232158E-30
0.2	4.132061E-31	1.004120E-30	1.564920E-30
0.3	1.597903E-31	5.760574E-31	1.015851E-30

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0.4	2.454373E-32	2.661642E-31	5.849511E-31
0.5	7.466508E-33	7.444043E-32	2.722206E-31
0.6	1.085586E-31	8.859188E-34	7.765935E-32
0.7	3.278200E-31	4.550071E-32	1.267426E-33
0.8	6.652507E-31	2.082848E-31	4.304481E-32
0.9	1.120851E-30	4.892382E-31	2.029915E-31

Table 4b- Table showing the Absolute Bias for Hussain and Shabbir (2007)RRT at $n = 100, x = 60, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.36445149	0.51224502	0.61464617
0.2	0.26445149	0.41224502	0.51464617
0.3	0.16445149	0.31224502	0.41464617
0.4	0.06445149	0.21224502	0.31464617
0.5	0.03554851	0.11224502	0.21464617
0.6	0.13554851	0.01224502	0.11464617
0.7	0.23554851	0.08775498	0.01464617
0.8	0.33554851	0.18775498	0.08535383
0.9	0.43554851	0.28775498	0.18535383

Figure 4a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 100, x = 60, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

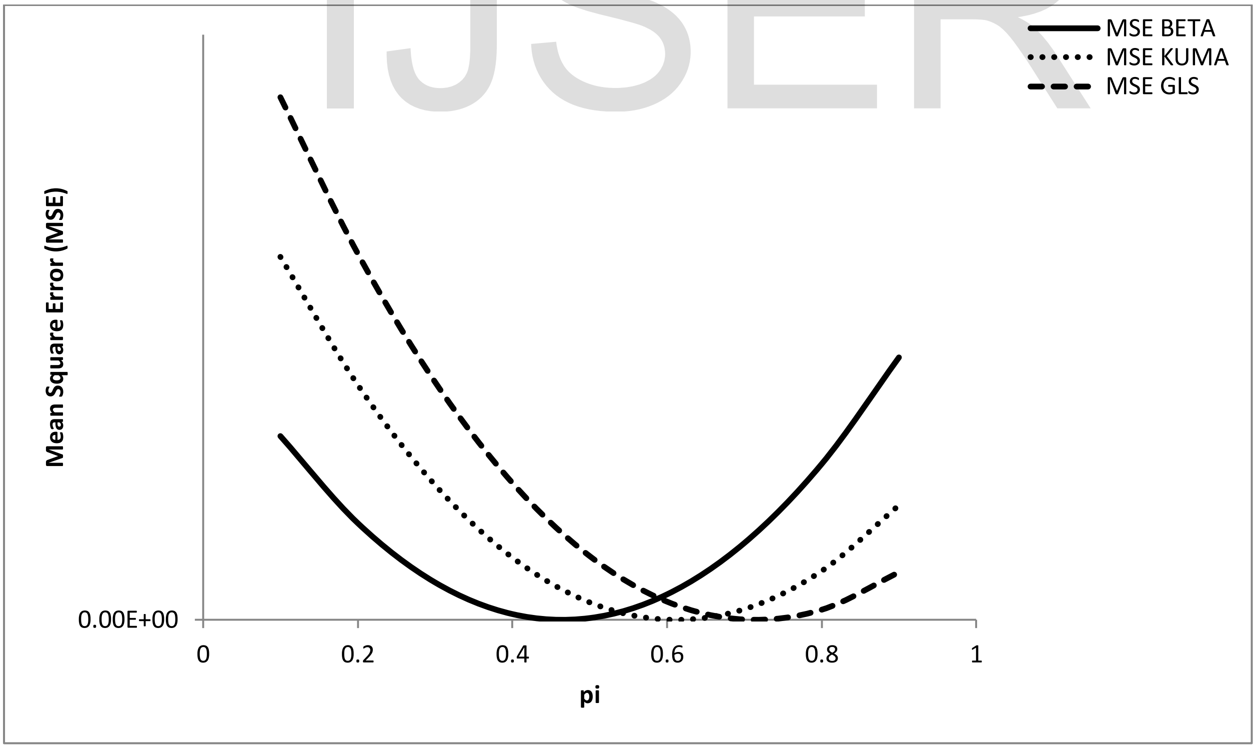

Figure 4b- Graph showing the Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 100, x = 60, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

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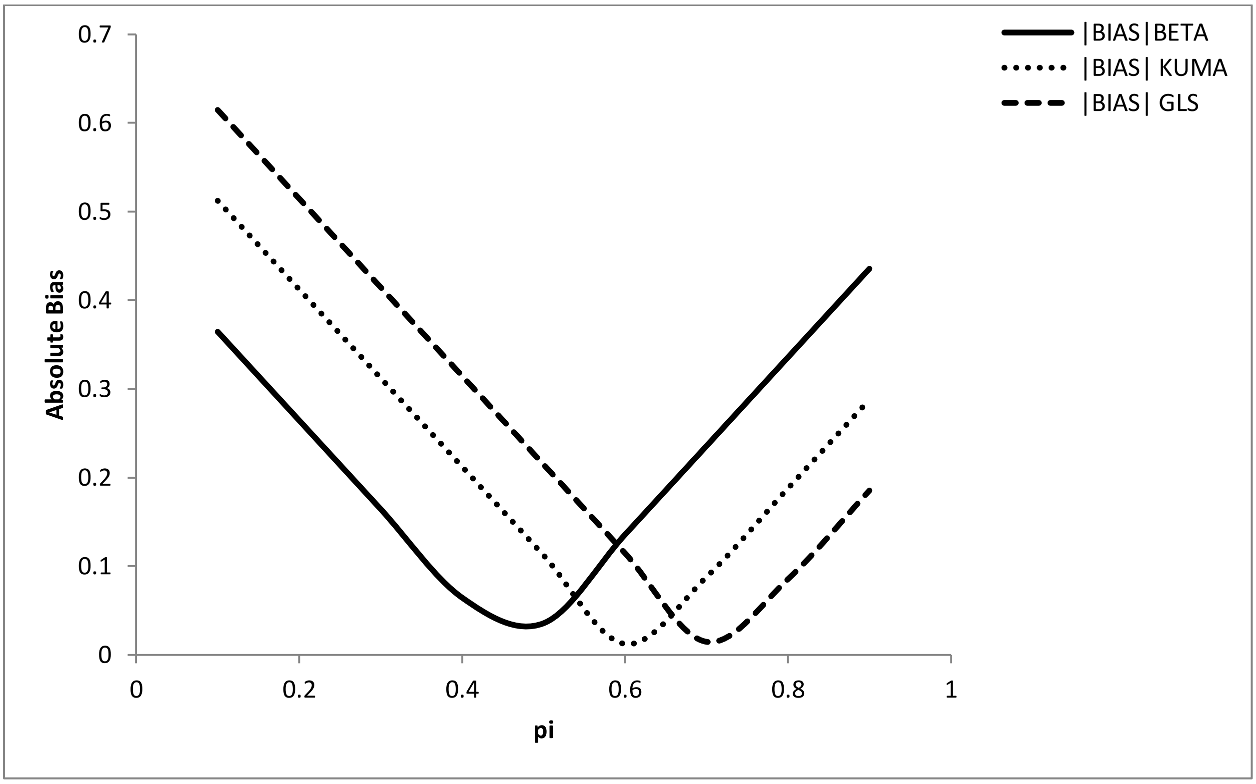

Comment: When $n = 100, P_1 = 0.1, 0.2$, the conventional estimator only perform on the average when π lies between 0.1 and 0.5 while the proposed estimators outperform the conventional estimator in obtaining higher and truthful responses from respondents with respect to stigmatized attribute when π lies between 0.6 and 0.9 respectively.

Table 5a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,$ $P_1 = 0.1,\ P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	1.291739E-74	1.566025E-74	1.830278E-74
0.2	7.144389E-75	9.218059E-75	1.126943E-74
0.3	3.068528E-75	4.473005E-75	5.933217E-75
0.4	6.898031E-76	1.425088E-75	2.294140E-75
0.5	8.214471E-78	7.430694E-77	3.522000E-76
0.6	1.023762E-75	4.206619E-76	1.073959E-76
0.7	3.736446E-75	2.464153E-75	1.559728E-75
0.8	8.146266E-75	6.204780E-75	4.709196E-75
0.9	1.425322E-74	1.164254E-74	9.555800E-75

Table 5b- Table showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,$ $P_2 = 0.9$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.39016110	0.42959180	0.46442454
0.2	0.29016110	0.32959180	0.36442454
0.3	0.19016110	0.22959180	0.26442454
0.4	0.09016110	0.12959180	0.16442454
0.5	0.00983890	0.02959180	0.06442454
0.6	0.10983890	0.07040820	0.03557546
0.7	0.20983890	0.17040820	0.13557546
0.8	0.30983890	0.27040820	0.23557546
0.9	0.40983890	0.37040820	0.33557546

Figure 5a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,\ P_2 = 0.9$

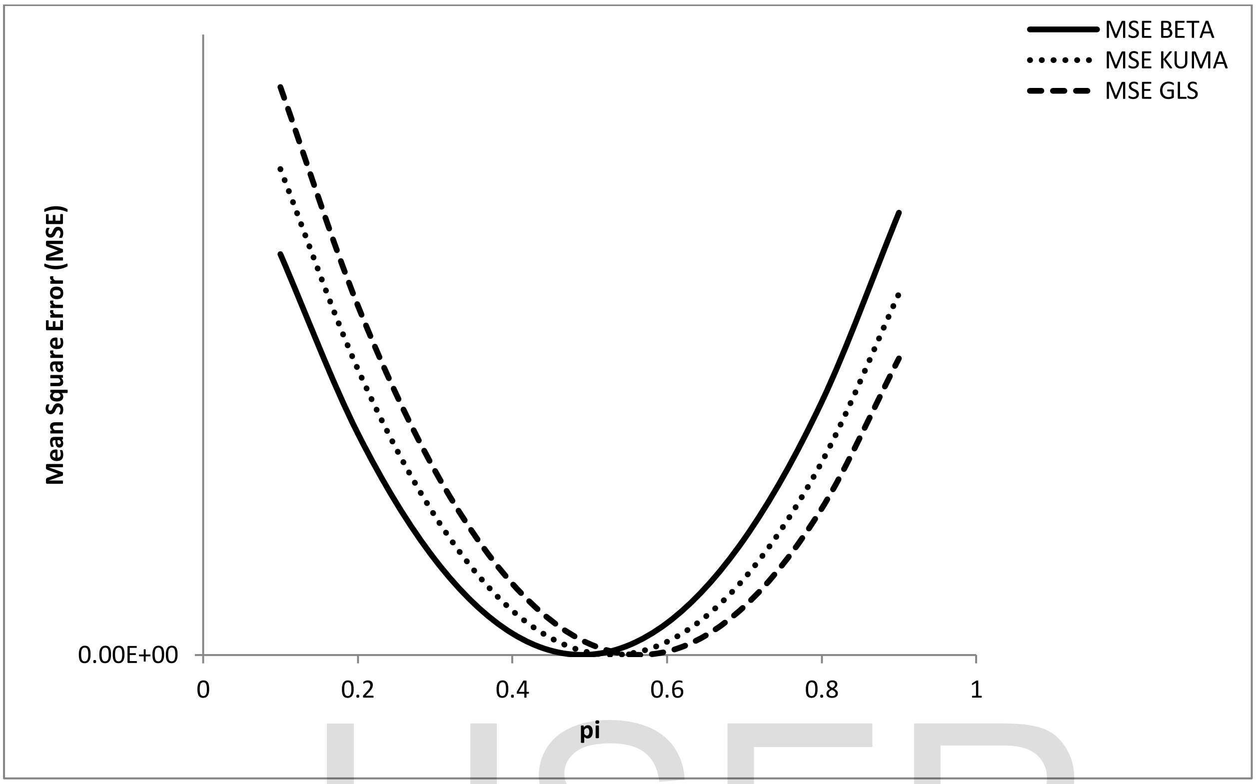

Figure 5b- Graph showing the Absolute Bias for Hussain and Shabbir [9] RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,\ P_1 = 0.1,$ $P_2 = 0.9$

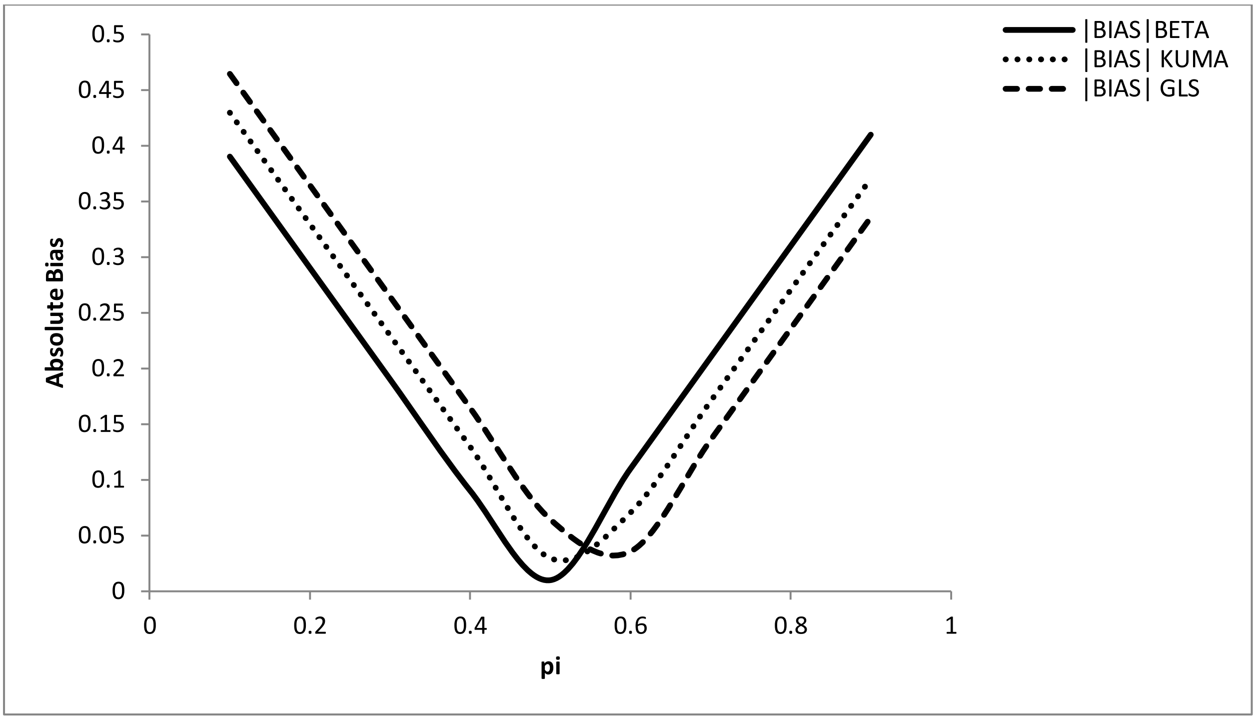

Table 6a- Table showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS

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0.1	1.248038E-74	1.717727E-74	2.161981E-74
0.2	6.820350E-75	1.039011E-74	1.390196E-74
0.3	2.857460E-75	5.300089E-75	7.881245E-75
0.4	5.917069E-76	1.907201E-75	3.557667E-75
0.5	2.308962E-77	2.114502E-76	9.312258E-76
0.6	1.151609E-75	2.128350E-76	1.920442E-78
0.7	3.977264E-75	1.911356E-75	7.697513E-76
0.8	8.500055E-75	5.307013E-75	3.234718E-75
0.9	1.471998E-74	1.039981E-74	7.396821E-75

Table 6b- Table showing the Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 250, x = 150, \alpha = 1,\ \beta = 10,\ P_1 = 0.2,$ $P_2 = 0.8$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.38350452	0.44991840	0.50475726
0.2	0.28350452	0.34991840	0.40475726
0.3	0.18350452	0.24991840	0.30475726
0.4	0.08350452	0.14991840	0.20475726
0.5	0.01649548	0.04991840	0.10475726
0.6	0.11649548	0.05008160	0.00475726
0.7	0.21649548	0.15008160	0.09524274
0.8	0.31649548	0.25008160	0.19524274
0.9	0.41649548	0.35008160	0.29524274

Figure 6a- Graph showing the Mean Square Errors (MSEs) for Hussain and Shabbir (2007) RRT at $n = 250, x = 150, \alpha = 1,$ $\beta = 10,\ P_1 = 0.2,\ P_2 = 0.8$

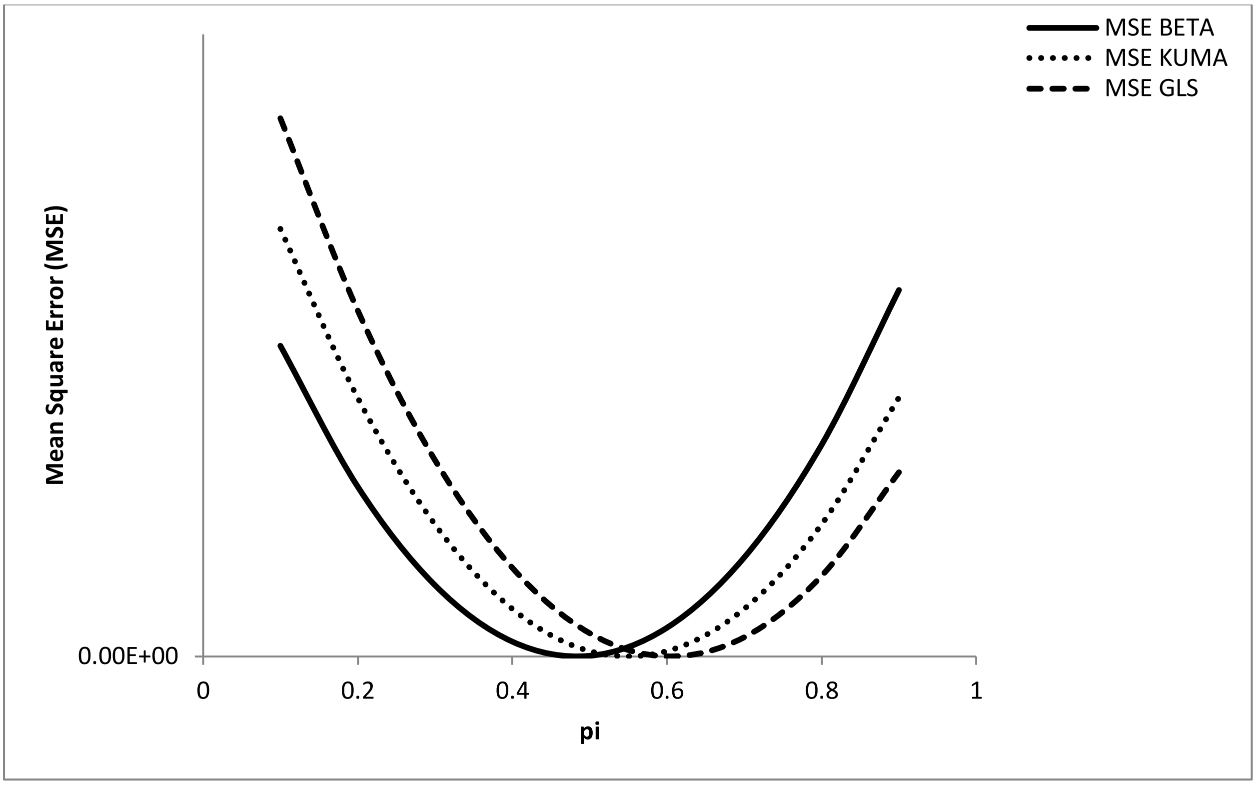
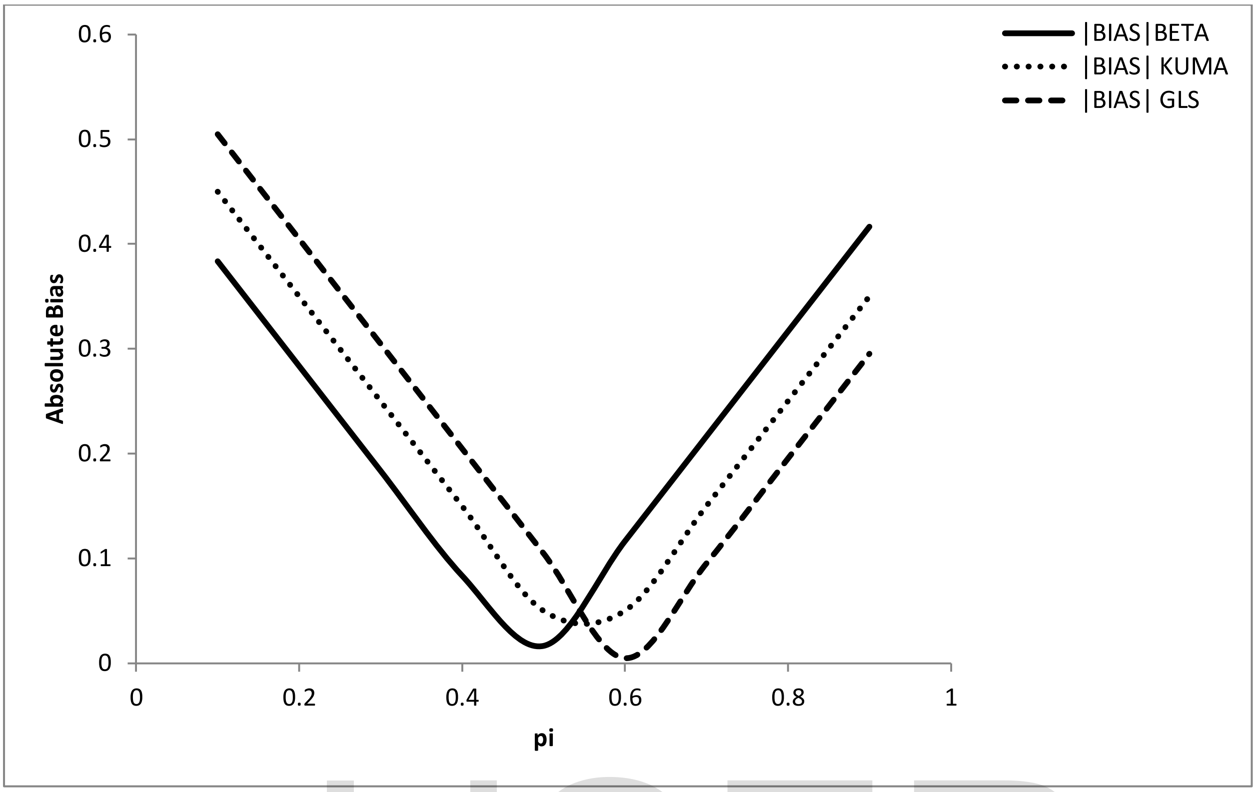


Figure 6b- Graph showing the Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 250, x = 150, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$



Comment: When $n = 250, P_1 = 0.1, 0.2$, the proposed estimators still perform better than the conventional estimator in obtaining higher and truthful responses from respondents with respect to stigmatized attribute when π lies between 0.6 and 0.9 as evident from the tables and figures above.

Discussions of Results

We can deduce from the results presented in tables and figures 1a to 6b that the newly developed alternative Bayesian estimators assuming both Kumaraswamy (KUMA) and Generalised (GLS) beta prior distributions as the family of alternative beta priors performed better than the usual Bayesian estimator with simple beta prior distribution proposed by Hussain and Shabbir [10] in obtaining responses from respondents possessing stigmatized attributes. Specifically, it can be seen from the results that the usual Bayesian estimator with simple beta prior only performed on the average as depicted in tables and figures presented above. The proposed Bayesian estimators are therefore better in obtaining higher responses from respondents in any survey which asks sensitive questions. This shows the superiority of the newly developed alternative Bayesian estimators in obtaining responses from respondents possessing stigmatized attributes.

5. Conclusion

We have developed the alternative Bayesian estimation of the population proportion when the data were collected through the Randomized Response Technique proposed by Hussain and Shabbir [9] assuming both Kumaraswamy (KUMA) and Generalised (GLS) Beta priors as the family of alternative beta prior distributions in addition to simple Beta prior distribution used by Hussain and Shabbir [10]. We presented our results in tables and figures for some pre-assigned values of the parameters and population proportion. We observed clearly that for relatively small, moderate as well as large sample sizes, the proposed estimators performed significantly better than that of Hussain and Shabbir [10].

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